C2 June 2007

1) $\int_1^8 x^{-\frac{1}{2}}\,dx = \Big[2x^{\frac{1}{2}}\Big]_1^8 = \Big[2\sqrt{x}\Big]_1^8$

$= 2\sqrt{8} - 2\sqrt{1}$

$= 2\sqrt{8} - 2$

$= -2 + 2\sqrt{2}\sqrt{4}$

$\underline{\underline{= -2 + 4\sqrt{2}}}$

2a) $f(x) = 3x^3 - 5x^2 - 16x + 12$

$(x-2) \therefore f(2)$

$f(2) = 3(2)^3 - 5(2)^2 - 16(2) + 12$

$\underline{\underline{= -16}}$

b)

$$\begin{array}{r l}
 & 3x^2 - 11x + 6 \\
(x+2) & \overline{)\,3x^3 - 5x^2 - 16x + 12} \\
 & 3x^3 + 6x^2 \\
 & \quad -11x^2 - 16x \\
 & \quad -11x^2 - 22x \\
 & \qquad 6x + 12 \\
 & \qquad 6x + 12 \\
 & \qquad \overline{\quad 0}
\end{array}$$

$(x+2)(3x^2 - 11x + 6)$

$x^2 - 11x + 18$

$\cancel{(x-3)(x}$

$(x-9)(x-2)$

$(x-3)(3x-2)$

$\boxed{(x+2)(x-3)(3x-2)}$

3a) $(1+kx)^6 = 1 + 6kx + \frac{6(6-1)}{2}(kx)^2 + \frac{6(6-1)(6-2)}{3!}(kx)^3$

$= 1 + 6kx + 15k^2x^2 + 20k^3x^3$

b) coeffs of x = x^2

$6k = 15k^2$

$6 = 15k$

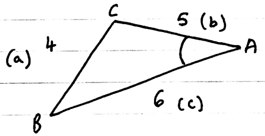
$k = \frac{6}{15} = \frac{2}{5}$

c) coeff of x^3 = $20k^3$

$\cancel{20\left(\frac{6}{15}\right)^3}$ $20\left(\frac{2}{5}\right)^3 = \underline{\underline{1.28}}$

4) cosine rule :

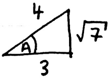
$a^2 = b^2 + c^2 - 2bc\cos A$



$\frac{a^2 - b^2 - c^2}{-2bc} = \frac{4^2 - 5^2 - 6^2}{-2\times5\times6} = \cos A$

$\cos A = \frac{3}{4}$

b) $\cos = C^a_h$



$\sin A = \frac{\sqrt{7}}{4}$

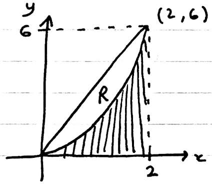
5) $y = x\sqrt{x^3+1}$

x	0	0.5	1	1.5	2
y	0	0.530	1.414	3.137	6

b) $\frac{1}{2} \times 0.5 \times \left[(0+6) + 2(0.530 + 1.414 + 3.137)\right]$

$= \underline{\underline{4.04}}$ (3.s.f)

c)



R = area triangle − integral

$= \frac{1}{2} \times 2 \times 6 - 4.04$

$= 6 - 4.04$

$= \underline{\underline{1.96}}$ (3.s.f)

6a) $8^x = 0.8$

$\log_8 0.8 = x$

$x = -0.107$ (3.s.f)

b) $2\log_3 x - \log_3 7x = 1$

$\log_3 x^2 - \log_3 7x = 1$

$\log_3 \left(\frac{x^2}{7x}\right) = 1$

$\frac{x}{7} = 3$

$\underline{\underline{x = 21}}$

7a) $\overrightarrow{MA}$ gradient $= \dfrac{y_1 - y_2}{x_1 - x_2} = \dfrac{1 - -2}{3 - 1} = \dfrac{3}{2}$

$M(\overset{x_1}{3}, \overset{y_1}{1})$

$A(\overset{x_2}{1}, \overset{y_2}{-2})$ m of l is perpendicular $= -\dfrac{2}{3}$

$$y - 1 = -\frac{2}{3}(x - 3)$$

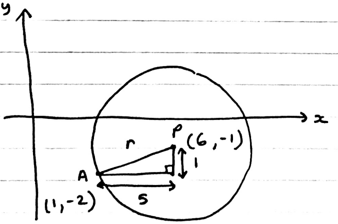
$$3y - 3 = -2x + 6$$

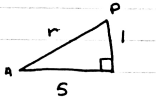
$$\underline{3y = -2x + 9}$$

b) $3y = -2x + 9$ when $x = 6$

$$y = \frac{-2(6) + 9}{3} = \frac{-3}{3} = \underline{\underline{-1}}$$

c) centre $= (6, -1)$





$r = \sqrt{5^2 + 1^2}$

$r = \sqrt{26}$

$\therefore r^2 = 26$

$$\boxed{(x - 6)^2 + (y + 1)^2 = 26}$$

8) $a = 50{,}000$

$r = r$

$u_n = 50{,}000r^{n-1}$

b)

$50{,}000r^{n-1} > 200{,}000$

$r^{n-1} > 4$

$\log r^{n-1} > \log 4$

$(n-1)\log r > \log 4$

$n-1 > \frac{\log 4}{\log r}$

$n > \frac{\log 4}{\log r} + 1$

c) $n > \frac{\log 4}{\log(1.09)} + 1$

$n > 16.086\ldots + 1$

$n > 17.086\ldots$ ← next positive whole integer.

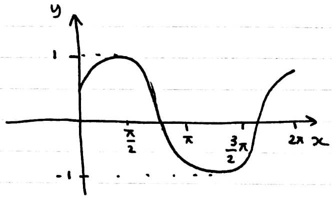
$\therefore$ year 18

d) 2006 – 2015 = 10 years

$S_{10} = \frac{a(1-r^{10})}{1-r} = \frac{50{,}000(1-(1.09)^{10})}{1-1.09} = 759646.48\ldots$ to nearest £10,000

$\boxed{= £760{,}000}$

9a) $y = \sin(x + \frac{\pi}{6}) \Rightarrow y = \sin x$ graph with a $\frac{\pi}{6}$ shift in -VE direct



b) meets x axis at $\pi - \frac{\pi}{6}$ and $2\pi - \frac{\pi}{6}$

$(\frac{5}{6}\pi, 0)$, $(\frac{11}{6}\pi, 0)$

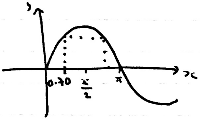
and y axis when $x = 0$ $y = \sin(\frac{\pi}{6}) = \frac{1}{2}$

$(0, \frac{1}{2})$

c) $\sin(x + \frac{\pi}{6}) = 0.65$

$x + \frac{\pi}{6} = \sin^{-1}(0.65) = 0.708..$

$\pi - 0.708 = 2.434..$



$x + \frac{\pi}{6} = 0.708, 2.434$

$\boxed{x = 0.18, 1.91}$ (2.d p)

10a) surface area $= 2 \times 2x^2 + 2 \times 2xy + 2xy$

$= 4x^2 + 6xy$

$600 = 4x^2 + 6xy$

$\frac{600 - 4x^2}{6x} = y$

$V = 2x^2 y = 2x^2\left(\frac{600 - 4x^2}{6x}\right)$

$V = \frac{1200x^2 - 8x^4}{6x} = 200x - \frac{4}{3}x^3$

b) $\frac{dV}{dx} = 200 - 4x^2$

$\frac{dV}{dx} = 0$ $\quad 200 = 4x^2$

$x^2 = 50$

$x = \pm\sqrt{50}$

$x = \sqrt{50}$

$V = 200(\sqrt{50}) - \frac{4}{3}(\sqrt{50})^3 \approx 943 \text{ cm}^3$

c) $\frac{d^2V}{dx^2} = -8x$

$-8x < 0$ $\quad -8\sqrt{50} < 0$

$\therefore \frac{d^2V}{dx^2} < 0$ $\therefore$ it is a maximum